DIGITAL CONTROL DESIGN BY THE POLYNOMIAL METHOD WITH EVALUATION OF THE SENSITIVITY FUNCTION

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Abstract— This paper presents two tuning design possibilities for an incremental digital RST controller based on the pole placement technique, whose parameters are obtained by solving the Diophantine equation. Since output and input disturbances occur in real systems, it is done the stability and robustness analysis of the control loop by means of the sensitivity functions loop shaping. To evaluate the controller performance for such tunings, it is analyzed the integral perfomance index for the error and control effort. The difference between the tuning designs is presented by numerical simulation for a underdamped second order system, showing that the proposed method can provide solutions with good performance.

Keywords— Reference tracking, disturbance rejection, sensitivity function, robustness analysis.

1 Introduction

Most of control systems design problems have specifications in terms of robustness, disturbance rejection and reference tracking. However, it is very difficult to achieve all these specifications using one degree of freedom controllers (Galdos et al., 2011). In this case, it is preferred to deal with two degree of freedom controllers, which allow treating disturbance rejection and reference tracking independently. It allows good performance for the system whilst preserving its robustness index when well tuned. In addition, these controllers assume the compromise that measurement errors do not generate excessive fluctuations in the control signal (Ostertag and Godoy, 2005).

Reference Signal Tracking (RST) controller design is the elegant pole placement based controller among the various methods available for linear SISO systems. Basically RST controller has a two degree of freedom structure and comprises of three polynomials namely R, S and T which are usually tuned by pole placement method, based on the solution of a Diophantine equation, which provides a control law that imposes a desired closed loop dynamic. It has become very useful in industrial applications. Moreover, system dynamics characteristics are transferred to the RST control law, allowing the achievement of two levels of performance in tracking and regulation (Landau, 1998).

The diophantine equation does not have a unique solution and different solutions of the RST controller parameters have different implications related to the desired performance specifications (Sung and Hara, 1988; Galdos et al., 2011). Thus, this paper presents two methods to tune R, S and T that provide the best performance for the closed loop system, keeping, for both cases, with the T fixed. It is made by evaluating the complementary sensitivity and sensitivity functions loop shaping. The first tuning is obtained by prioritizing R with respect to S and the second tuning is obtained by prioritizing S with respect R.

Although the pole placemente technique is already consolidated in the control area, this article appears as a contribution of the exploration of the pole placement technique by showing among several possible solutions for the Diophatne equation, two ways of tuning the polynomials R and S for systems without integrators that provide better performance for reference tracking and disturbance rejection. The analysis of robustness of the tunings is made by using the complementary sensitivity and sensitivity functions.

Finally, simulations are shown to evaluate the benefits of this method in terms of the reference tracking, as well as for the regulation in the presence of disturbances. Highlights are made for plants without integrators and because of this, design model augmentation by integrator addition is used (Landau and Zito, 2005).

2 RST Canonical Structure

According to Figure 1, it is assumed that a discrete time process, with control signal $u(k)$, measured output signal $y(k)$ and input and output disturbances $v(k)$ and $p(k)$, respectively, is described by (Franklin et al., 2013)

$$y(k) = \frac{z^{-1}B(z^{-1})}{A(z^{-1})}u(k) + \frac{z^{-1}B(z^{-1})}{A(z^{-1})}v(k) + p(k) \tag{1}$$

where

$$\begin{cases} A(z^{-1}) = 1 + a_1 z^{-1} + ... + a_{na} z^{-na} \\ B(z^{-1}) = b_0 + b_1 z^{-1} + ... + b_{nb} z^{-nb} \end{cases} \tag{2}$$

As shown in Figure 1, the RST controller is structured by $R\left(z^{-1}\right)$, $S\left(z^{-1}\right)$ and $T\left(z^{-1}\right)$. By

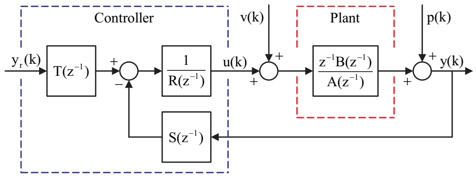

Figure 1: RST digital controller canonical form.

assuming a priori $v(k)$ and $p(k)$ to be null, it provides a control law $u(k)$ with the following form:

$$R(z^{-1})u(k) = T(z^{-1})y_r(k) - S(z^{-1})y(k) \quad (3)$$

where $R\left(z^{-1}\right)$, $S\left(z^{-1}\right)$ and $T\left(z^{-1}\right)$ are weighting polynomials of the control, system output and reference signals, respectively. The choice of these polynomials allows solving the problem of regulation and reference tracking and are given by

$$\begin{aligned} R(z^{-1}) &= r_0 + r_1 z^{-1} + r_2 z^{-2} + \ldots + r_{nr} z^{-nr} \\ S(z^{-1}) &= s_0 + s_1 z^{-1} + s_2 z^{-2} + \ldots + s_{ns} z^{-ns} \\ T(z^{-1}) &= t_0 + t_1 z^{-1} + t_2 z^{-2} + \ldots + t_{nt} z^{-nt} \end{aligned} \quad (4)$$

According to Landau and Zito (2005) there are two forms of getting RST controller: Positional RST and Incremental RST. The Positional RST structure is represented by Equation 3 and does not guarantee reference tracking and disturbance rejection for systems without integrators, while the incremental RST does.

Therefore, in order to satisfy the desired performance specifications for systems without integrators and to guarantee reference tracking and disturbance rejection, integral control action must be added, which for digital controllers is called incremental action. The discrete time representation of the incremental action of the RST controller is obtained by including Δ in the Equation 3, such a shows the Equation 5.

$$\Delta u(k) = \frac{T(z^{-1})}{R(z^{-1})} y_r(k) - \frac{S(z^{-1})}{R(z^{-1})} y(k) \quad (5)$$

Since $\Delta = 1 - z^{-1}$ acts on the controller output giving it an incremental action given by

$$\Delta u(k) = u(k) - u(k-1) \quad (6)$$

the control signal applied to the plant is

$$u(k) = u(k-1) + \Delta u(k) \quad (7)$$

For design purposes and to compensate the Equation 5, the Equation 1 is assumed to be given by

$$y(k) = \frac{z^{-1}B(z^{-1})}{\Delta A(z^{-1})} \Delta u(k) \quad (8)$$

Remaking the Equation 8, the augmented model of the plant is given by Equation 9.

$$\Delta A(z^{-1}) y(k) = z^{-1} B(z^{-1}) \Delta u(k) \quad (9)$$

Where $\Delta A\left(z^{-1}\right) = \left(1 - z^{-1}\right) A\left(z^{-1}\right)$. The Equation 9 represents the augmented system by incremental action. This assumption ensures reference tracking and disturbance rejection. Therefore, the incremental RST controller design by pole placement technique is done based on the Equation 9.

It is important to emphasize that the model that represents the plant does not change. The new mathematical formulation in Equation 9 was done only to add the incremental action and to allow the design of the incremental RST. In addition, the control signal that will be applied to the plant is that shown in Equation 7. Therefore, the closed loop system is obtained by including the Equation 5 into Equation 9, given

$$\frac{y(k)}{y_r(k)} = \frac{z^{-1}B(z^{-1})T(z^{-1})}{R(z^{-1})\Delta A(z^{-1}) + B(z^{-1})S(z^{-1})} \quad (10)$$

3 Pole Placement Technique

This section presents the pole placement technique in the case in which no process zeros are cancelled and the aim is to determine the coefficients of the RST controller that gives desired closed-loop poles for the system. In addition, it is required that the system follows command signals in a specified form. This is a simple method that, properly applied, can give practically useful controllers for systems without integrators (Åström and Wittenmark, 2013).

The RST controller is tuned according to pole placement technique presented in Åström and Wittenmark (2013). This technique is comprised of a dynamic compensator, where the feedback data comes from an observer system, avoiding pole and zero cancelations, ensuring greater robustness.

Thereby, since no process zero is cancelled, the desired closed loop poles are then composed by two parts: one that corresponds to the controller $H_c\left(z^{-1}\right)$ and another that corresponds to the observer dynamic $H_o\left(z^{-1}\right)$ (Åström and Wittenmark, 2008). The closed loop system zeros are composed by the open loop zeros of $B\left(z^{-1}\right)$ and the feed forward part in $T\left(z^{-1}\right)$ to compensate the static gain and to guarantee offset free between reference and output signal. Thus, the closed-loop system with desired performance is

$$\frac{y(k)}{y_r(k)} = \frac{z^{-1}B(z^{-1})T(z^{-1})}{H_c(z^{-1})H_o(z^{-1})} \quad (11)$$

In Equation 11, the polynomial $H_c\left(z^{-1}\right)$ is the polynomial that contains the desired poles and $H_o\left(z^{-1}\right) = A\left(z^{-1}\right)$ is the observer polynomial. Such polynomials are given by

$$H_c(z^{-1}) = 1 + d_1 z^{-1} + ... + d_{nd} z^{-nd} \quad (12)$$

$$H_o(z^{-1}) = 1 + a_1 z^{-1} + ... + a_{na} z^{-na} \quad (13)$$

The desired closed loop polynomial is

$$H(z^{-1}) = H_c(z^{-1}) H_o(z^{-1}) \quad (14)$$

According to pole placement technique, the Equations 10 and 11 must be the same, leaving to the diophatine equation summarized by

$$\begin{aligned} H_c(z^{-1})H_o(z^{-1}) = R(z^{-1})\Delta A(z^{-1}) \\ +B(z^{-1})S(z^{-1}) \end{aligned} \quad (15)$$

In a dynamic compensator it is also natural to introduce the command signals in such way that it does not generate observer errors. It is done by considering $T\left(z^{-1}\right)$ being

$$T(z^{-1}) = T_{off} H_o(z^{-1}) \quad (16)$$

where $T_{off} = \frac{H_c(1)}{B(1)}$. That is, when $z \to 1$, which is the static contribution of these polynomials. The parameter T_{off} is a parameter that compensates the error between reference and plant output signal by static approach (Åström and Wittenmark, 2008).

The polynomials $R\left(z^{-1}\right)$ and $S\left(z^{-1}\right)$ can be obtained by solving the Equation 15. Since the Diophantine equation do not provides a unique solution and different solutions of the RST parameters provide different performance related to the reference tracking and disturbance rejection, this paper proposes two tunings for $R\left(z^{-1}\right)$ and $S\left(z^{-1}\right)$, keeping $T\left(z^{-1}\right)$ as in Equation 16. These two tuning guarantee good performance for reference tracking and disturbance rejection.

3.1 RST tuning prioritizing $R\left(z^{-1}\right)$ polynomial

This tuning is achieved by considering $nr = 1$ and $ns \leq 2$. This condition satisfies the Equation 15. Therefore, the RST controller is characterized by

$$\begin{aligned} R(z^{-1}) &= r_0 + r_1 z^{-1} \\ S(z^{-1}) &= s_0 + s_1 z^{-1} + ...s_{ns} z^{-ns} \\ T(z^{-1}) &= t_0 + t_1 z^{-1} + ... + t_{nt} z^{-nt} \end{aligned} \quad (17)$$

3.2 RST tuning prioritizing $S\left(z^{-1}\right)$ polynomial

Different from the first tuning, this second tuning is achieved by considering $nr = 0$ and $ns \geq 1$. That condition satisfies the Equation 15. Therefore, the RST controller is characterized by

$$\begin{aligned} R(z^{-1}) &= r_0 \\ S(z^{-1}) &= s_0 + s_1 z^{-1} + ...s_{ns} z^{-ns} \\ T(z^{-1}) &= t_0 + t_1 z^{-1} + ... + t_{nt} z^{-nt} \end{aligned} \quad (18)$$

It is important to emphasize that the RST tuning by prioritizing $R\left(z^{-1}\right)$ polynomial is more recommended for systems whose desired closed loop response is of second order regardless of system order. On the other hand, the second tuning is recommended for systems whose desired closed loop response is of first order regardless of system order too, because it satisfies the diophantine equation in 15.

4 Sensitivity Functions

Usually, sensor measurement noises and modeling errors are unmodeled high frequency dynamics. Reference signals and load disturbances are low frequency dynamics. Since these effects can often cause unsatisfactory behaviour in control loops that was designed without taking them into account, it is important to have controllers that guarantee performance and robustness despite of these undesirable effects (Åström and Wittenmark, 2013).

Thus, this work covers robustness analysis using sensitivity functions magnitude plots in the frequency domain. It is used to provide measures of how sensitive the closed loop system is to changes in the plant. From these functions the Gain Margin (GM) and Phase Margin (PM) are obtained to quantify the trade off between robustness and performance to guarantee a suitable well tuned controller (Seborg et al., 2010; Åström and Wittenmark, 2013).

The complementary, input and output sensitivity functions are, respectively,

$$T(z^{-1}) = \frac{z^{-1}B(z^{-1})T(z^{-1})}{R(z^{-1})\Delta A(z^{-1}) + B(z^{-1})S(z^{-1})} \quad (19)$$

$$S_i(z^{-1}) = \frac{z^{-1}B(z^{-1})R(z^{-1})}{R(z^{-1})\Delta A(z^{-1}) + B(z^{-1})S(z^{-1})} \quad (20)$$

$$S_o(z^{-1}) = \frac{R(z^{-1})\Delta A(z^{-1})}{R(z^{-1})\Delta A(z^{-1}) + B(z^{-1})S(z^{-1})} \quad (21)$$

The input sensitivity function $S_i\left(z^{-1}\right)$ characterizes the effect of a disturbance $v\left(k\right)$ acting on the plant input, whereas the output sensitivity function $S_o\left(z^{-1}\right)$ characterizes the effect of a disturbance $p\left(k\right)$ acting on the plant output (see Figure 1).

Both are shaped for disturbance rejection analysis. The complementary sensitivity $T\left(z^{-1}\right)$ is equivalent to the closed loop transfer function for set point changes and it is shaped for reference tracking analysis.

In a SISO (Single Input Single Output) sense, for achieving the desired reference tracking and disturbance rejection, $S_o\left(z^{-1}\right)$ must be kept small ($\left|S_o\left(e^{j\omega T_s}\right)\right| \to 0$) and the complementary sensitivity $T\left(z^{-1}\right)$ must be kept unit value ($\left|T\left(e^{j\omega T_s}\right)\right| \to 1$) at low frequencies, while at high frequencies the absolute value of the $S_o\left(z^{-1}\right)$ must go to unity ($\left|S_o\left(e^{j\omega T_s}\right)\right| \to 1$) and $T\left(z^{-1}\right)$ must be kept bounded ($\left|T\left(e^{j\omega T_s}\right)\right| \to 0$) for guaranteed good sensor noise suppressing ability (Doyle et al., 1990; Seborg et al., 2010).

The input sensitivity function $S_i\left(z^{-1}\right)$ must be kept small ($\left|S_i\left(e^{j\omega T_s}\right)\right| \to 0$) at low and high frequencies (Seborg et al., 2010; Deepika and Narayan, 2015). Robustness analysis is done using the maximum amplitude ratios of the complementary sensitivity M_T and output sensitivity functions M_S, which are defined by

$$M_T \triangleq \max_{\omega} \left|T(e^{j\omega T_s})\right| \tag{22}$$

$$M_S \triangleq \max_{\omega} \left|S_o(e^{j\omega T_s})\right| \tag{23}$$

These variables are used to quantify the sensibility of the control loop to the excitation signals under consideration. It means that small M_S values make the system less sensible to input or output disturbance $p\left(k\right)$, whereas M_T considers the influence of the reference signal $y_r\left(k\right)$, and it is equivalent to the amplitude of the resonant peak as well, that in general, is desirable to be kept small (Seborg et al., 2010).

According to Seborg et al. (2010), M_S should be in the range of 1.2 to 2 and M_T should be in the range of and 1.0 to 1.5. These requirements provide good GM and PM for the closed loop system and maintain the compromise between robustness and performance. GM and PM are obtained by

$$GM_S \geq 20\log 10\left(\frac{M_s}{M_S - 1}\right) \tag{24}$$

$$PM_S \geq 2\sin^{-1}\left(\frac{1}{2M_S}\right)\left(\frac{180}{\pi}\right) \tag{25}$$

$$GM_T \geq 20\log 10\left(1 + \frac{1}{M_T}\right) \tag{26}$$

$$PM_T \geq 2\sin^{-1}\left(\frac{1}{2M_T}\right)\left(\frac{180}{\pi}\right) \tag{27}$$

The Gain Margin is the amount that the loop gain can be increased before reaching the stability limit, while the Phase Margin is the amount of phase lag required to reach the stability limit (Ogata, 2010; Stevens et al., 2015).

5 Results

Consider a second order under-damped system with a sampling period $T_s = 0.05$ s:

$$\frac{y(k)}{u(k)} = \frac{z^{-1}(-0.001785 + 0.1864z^{-1})}{1 - 1.664z^{-1} + 0.8558z^{-2}} \tag{28}$$

The desired closed loop response has a damping coefficient $\xi_d = 0.9$ and natural frequency $\omega_n = 10$ rad/s. Thus $H_c(z^{-1})$ and $H_o(z^{-1})$ are

$$H_c(z^{-1}) = 1 - 1.2451z^{-1} + 0.4066z^{-2} \tag{29}$$

$$H_o(z^{-1}) = 1 - 1.664z^{-1} + 0.8558z^{-2} \tag{30}$$

The obtained results of the closed loop system for both tuning of the RST controller are shown in Figure 2 to 6.

In order to quantitatively and qualitatively evaluate the performance deviation of the system control loop when it is controlled by RST controller, the ISE (Integral Square Error) performance index was chosen, where $e(k)$ is the difference between the reference $y_r(k)$ and the measured output $y(k)$, calculated by:

$$ISE = \sum_{k=1}^{n} e(k)^2 = \left(y_r\left(k\right) - y\left(k\right)\right)^2 \tag{31}$$

It was also chosen to evaluate the control effort $u(k)$ of the controller the TVC (Total Variation Control) performance index, calculated by:

$$TVC = \sum_{k=1}^{n} \left|u\left(k\right) - u\left(k-1\right)\right| \tag{32}$$

In the Figure 2 at the instant of time $t = 1$ s is applied a unit step input to the system and at the instant of time $t = 5$ s is applied a load disturbance $v(k)$ at the plant input with magnitude of 0.5. In the Figure 3 at the instant of time $t = 1$ s is applied a unit step input to the system and at the instant of time $t = 5$ s is applied a load disturbance $p(k)$ at the plant output with magnitude of 0.5.

It is verified that the two controller tunings provide good performances for reference tracking, however for disturbance rejection the RST controller tuning prioritizing $R\left(z^{-1}\right)$ provides a better performance than the RST controller tuning prioritizing $S\left(z^{-1}\right)$. It is concluded by the calculated performance index ISE and TVC presented in Figure 2 and 3. The Bode diagram of the open loop and closed loop system is presented in Figure 4. The closed loop system corresponds to the complementary sensitivity function $T(z^{-1})$, which guarantees reference tracking (see Equation 19).

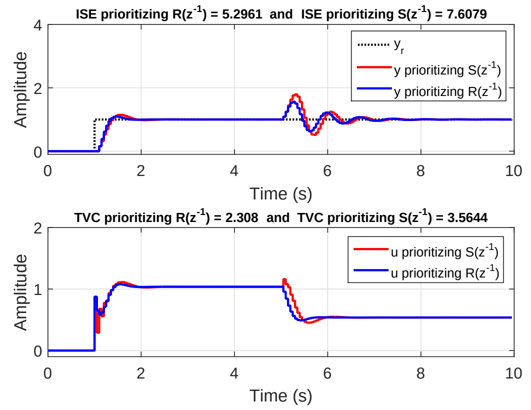

Figure 2: Step response with input disturbance.

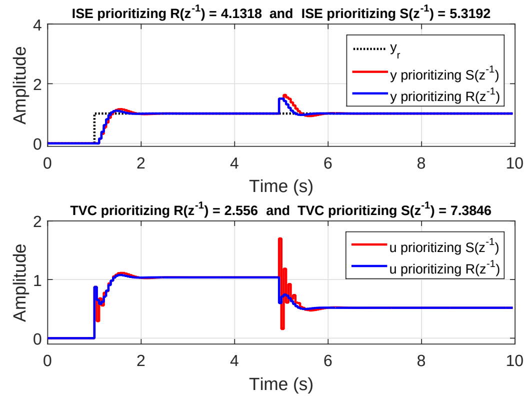

Figure 3: Step response with output disturbance.

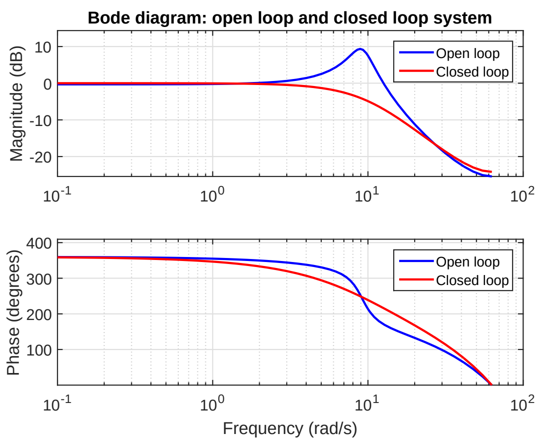

Figure 4: Bode diagram.

The figures 5 and 6 present, respectively, the shape of frequency response of the input and output sensitivity functions $S_i(z^{-1})$ and $S_o(z^{-1})$ draw by equations 20 and 21. Both controller tunings provide satisfactory disturbance rejection, moreover the incremental RST controller tuned by prioritizing $R(z^{-1})$ is less sensible to input or output disturbance than the incremental RST controller tuned by prioritizing $S(z^{-1})$. Therefore, the figures 5 and 6 justify the better disturbance rejection of the RST controller tuned by prioritizing $R(z^{-1})$ presented in figures 2 and 3.

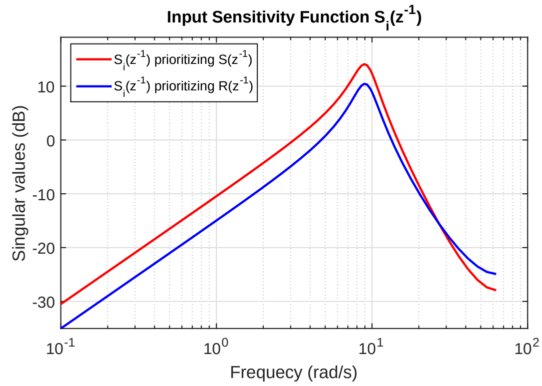

Figure 5: Input sensitivity function.

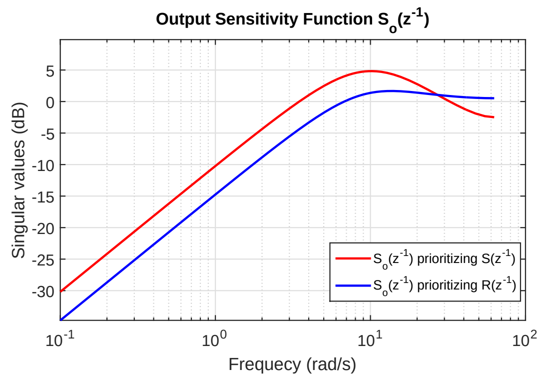

Figure 6: Output sensitivity function.

The robustness analysis is presented in tables 1 and 2. In the Table 1 are available the Gain Margin and Phase Margin of the complementary, input and output sensitivity functions when the RST controller is tuned prioritizing $R(z^{-1})$.

In turn, in the Table 2 are available the Gain Margin and Phase Margin of the complementary, input and output sensitivity functions when the RST controller is tuned prioritizing $S(z^{-1})$.

Table 1: Robustness analysis of the RST controller tuning prioritizing $R(z^{-1})$.

Function	**Gain Margin**	**Phase Margin**
$T(z^{-1})$	GM = 6.02 dB	PM = 60°
$S_i(z^{-1})$	GM = 1.23 dB	PM = 7.5°
$S_o(z^{-1})$	GM = 15.14 dB	PM = 48.7°

In the Table 3 are shown the maximum singular values of the complementary, input and output sensitivity functions when the RST controller is tuned prioritizing $R(z^{-1})$ and $S(z^{-1})$. It is noted that M_T for both tunings are the same, which explains the identical closed loop response. On the

Table 2: Robustness analysis of the RST controller tuning prioritizing $S(z^{-1})$.

Function	**Gain Margin**	**Phase Margin**
$T(z^{-1})$	GM = 6.02 dB	PM = 60°
$S_i(z^{-1})$	GM = 0.78 dB	PM = 4.9°
$S_o(z^{-1})$	GM = 7.41 dB	PM = 33.3°

other hand, the M_{S_i} and M_{S_o} with the RST controller tuning by prioritizing $R(z^{-1})$ are smaller than the M_{S_i} and M_{S_o} with the RST controller tuning by prioritizing $S(z^{-1})$, which explains the better disturbance rejection of the RST controller tuned prioritizing $R(z^{-1})$ in Figure 2 and 3.

Table 3: Maximum singular values of $T(z^{-1})$, $S_i(z^{-1})$ and $S_o(z^{-1})$.

Function	$R(z^{-1})$ **tuning**	$S(z^{-1})$ **tuning**
$T(z^{-1})$	$M_T = 1.00$	$M_T = 1.00$
$S_i(z^{-1})$	$M_{S_i} = 7.5705$	$M_{S_i} = 11.5743$
$S_o(z^{-1})$	$M_{S_o} = 1.2120$	$M_{S_o} = 1.7423$

6 Conclusions

In this paper, it was presented a very simple idea to solve the Diophantine equation in order to achieve a RST controller capable to deal better with input and output disturbances in control loop. Two different approaches to design a RST controller that was presented in section 3 provided better performance for reference tracking and disturbance rejection for systems without integrators, according to the section 4, which presented the stability and robustness analysis of the controller by using the sensitivity functions for both designs.

From numerical simulations of a second order under-damped system, performance index ISE and TVC calculated in the time response and Gain Margin and Phase Margin of the sensitivity functions calculated in the frequency response presented in this paper, it was concluded that the RST controller designed prioritizing $R(z^{-1})$ provided a better disturbance rejection with lower control effort and also it was guaranteed bigger Gain and Phase Margin when compared to the controller designed prioritizing $S(z^{-1})$ and the two approaches provide the same reference tracking for the control loop.

Acknowledgements

The authors are grateful for the financial support of the Post-Graduate Program in Electrical Engineering (PPGEE) of the Federal University of Pará (UFPA), CNPq and CAPES.

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