A ROBUST APPROACH FOR THE DIGITAL PD SYNTHESIS BY USING INTERVAL PARAMETRIC CONTROL

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Abstract— This paper presents the design of a proportional-derivative controller (PD controller) by means of the theory of polynomials with interval roots, and applies them to the problem of robust pole placement technique for an interval system represented by a DC motor. It is formulated a optimization problem by Linear Programming approach integrated with the Chebyshev theorem, which incorporates additional constraints on the system and desired performance parameters and allow the designer to find the controller parameters that place closed-loop poles within desired intervals for plants with parameter uncertainties. The design purpose is the minimization of the overall deviation from the desired performance for the closed loop system, as specified by a characteristic polynomials family. For performance comparison, it was designed a classic PD controller and the results shows the good performance.

Keywords— Linear Programming, Chebyshev Theorem, Robust Pole Placement, Robust Control

1 Introduction

Throughout the evolution of the studies of closed loop systems, the Control Theory has been providing tools for solving problems in various fields of humanity. Within the context of solutions for process control, one of the areas that have received great research efforts from the scientific community is Robust Parametric Control (RPC) Theory, that emerges as a set of modern control techniques whose objective is to avoid the negative effects caused by the uncertainties present in the system parameters (Barmish and Jury, 1994). The RPC gained greater attention from academia since the 1980s and this academic "boom" was originate as a result of Kharitonov's work, who developed the so-called seminal Kharitonov stability Theorem. Among the several methods of solution to the problem of controlling plants with parametric uncertainties, in the present paper, are highlighted the use of Linear Programming (LP) approach (Keel and Bhattacharyya, 1999) and the Chebyshev Theorem (Boyd and Vandenberghe, 2004). This methods, when combined, serve to design optimal and robust controllers.

In this paper, a design trend is presented for Proportional-Integral-Derivative (PID) controllers family and is based on parametric robust control theory. The controller designed is applied for asymptotic tracking of a motor DC and is expected to reduce the control effort when the system is operating outside its nominal operating point. The motivation arises from the difficulty of relating parametric robust control theory to conventional design methods. In conventional methods, the goal is to design a controller with fix parameters from a plant with fix parameters too. However, robust parametric control presents a different form of controller development, in which the modelling of the systems is developed with the plant parameters represented by real intervals (not fix). From the method presented by Keel and Bhattacharyya (1999), it is possible to develop controllers designs in the interval domain and still with fix parameters. The reason for choosing the controllers of the PID family is justified by the fact that it is the most popular controller in the industrial environment (Åström and Hägglund, 2006).

In the context of the work involving robust parametric control theory and PID controllers family, several theoretical and practical contributions can be cited. One can cite as an example the works of Keel and Bhattacharyya (1997) and Keel and Bhattacharyya (1999), in which a linear programming approach was used to provide necessary conditions on the fixed order controller design for robust stability of the closed loop systems. In Cunha et al. (2016), the design of a robust controller via linear programming is presented in comparison to a conventional design technique applied to the position control of a motor DC. In both works, only linear programming approach is used to solve the optimization problem proposed for optimal tuning of the controllers parameters. In de França Silva et al. (2016), the design and evaluation of an robust interval controller applied to the speed regulation problem of a synchronous generator is presented.

Following the same line of work cited and in view of the practical and theoretical contributions of the RPC combined with conventional control techniques, in this paper, the linear programming approach presented in Keel and Bhattacharyya (1999) is combined with the Chebyshev Theorem, given in Boyd and Vandenberghe (2004), for optimal tuning of the robust controller parameters, providing better performance, besides that, the robust controller design with robust stability guaranteed according to Kharitonov Theorem is presented. Another difference presented in the design developed in this paper is the possibility of synthesizing digital controllers through a digital mask for analogical controllers, ensuring lower degradation of the digital controller performance designed. One advantage of this approach is that it allows us to pick a controller that is itself robust.

2 Robust Stability Analysis

A system with parametric uncertainties is generally described by uncertain polynomials $N(s,n)$ and $D(s,d)$, restricted within pre-specified closed real intervals, according to (1) (Barmish and Jury, 1994; Bhattacharyya and Keel, 1995).

$$G(s,n,d) = \frac{N(s,n)}{D(s,d)} = \frac{\sum_{i=0}^{m} \left[{n_i}^{-}, {n_i}^{+}\right] s^i}{\sum_{i=0}^{n} \left[{d_i}^{-}, {d_i}^{+}\right] s^i} \quad (1)$$

Many robust stability tests under parametric uncertainty are based on analysis of uncertain characteristic polynomial assumed as a interval polynomial family (Barmish and Jury, 1994), such as

$$P(s,a) = \sum_{i=0}^{n} \left[{a_i}^{-}, {a_i}^{+}\right] s^i \quad (2)$$

According to (2), it is noted that the polynomial $P(s,a)$ is stable if all its roots remain contained on the left hand side of the complex plane. Then, $P(s,a)$ is robustly stable if all its polynomials are stable for a set of operating point different from nominal operation point, since it respecting their minimum and maximum limits (Keel and Bhattacharyya, 1997). However, instead of checking stability of an infinite number of polynomials we just have to check stability of four polynomials, which can be made using the Kharitonov Theorem.

2.1 Kharitonov Stability Theorem

The Kharitonov Theorem deals with the robust stability analysis of uncertain (interval) polynomials. In particular, it gives a computationally feasible algorithm for testing of stability by means of four fixed polynomials if the roots of $P(s,a)$ remain contained on the left hand side of the complex plane (Barmish and Jury, 1994).

Thus, an interval polynomial family $P(s,a)$ with invariant degree is robustly stable if and only if its four Kharitonov polynomials, given in (3), are stable (Barmish and Jury, 1994; Bhattacharyya and Keel, 1995)(i.e. they have all their roots in the left hand side of the complex plane).

$$\begin{aligned}
K_1(s) &= a_0^- + a_1^- s + a_2^+ s^2 + a_3^+ s^3 + a_4^- s^4 + a_5^- s^5 + a_6^+ s^6 + ...\\
K_2(s) &= a_0^+ + a_1^+ s + a_2^- s^2 + a_3^- s^3 + a_4^+ s^4 + a_5^+ s^5 + a_6^- s^6 + ...\\
K_3(s) &= a_0^+ + a_1^- s + a_2^- s^2 + a_3^+ s^3 + a_4^+ s^4 + a_5^- s^5 + a_6^- s^6 + ...\\
K_4(s) &= a_0^- + a_1^+ s + a_2^+ s^2 + a_3^- s^3 + a_4^- s^4 + a_5^+ s^5 + a_6^+ s^6 + ...
\end{aligned} \quad (3)$$

2.2 Robust Pole Placement Design

In this paper, the robust controller design uses two different procedures. The first is the tool developed in Keel and Bhattacharyya (1999), associated with a linear goal programming formulation, which will lead to a set linear inequality constraints. The second procedure is the Chebyshev theorem, developed in Boyd and Vandenberghe (2004), which provides a maximum stability region, characterized by a ball of center x_c and radium R, whose norm is Euclidean.

Thus, it is possible obtain a controller $C(s)$ of order r, given in (4), able to ensure the robust stability of the uncertain systems $G(s,q)$ of order n, given in (5), according to Figure 1, where y_r, u and y are, respectively, the reference signal, the control signal and the system output.

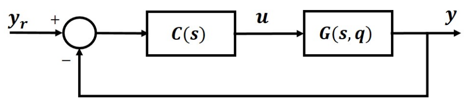

Figure 1: General system structure

$$G(s,q) := \frac{n_n s^n + n_{n-1} s^{n-1} + ... + n_0}{d_n s^n + d_{n-1} s^{n-1} + ... + d_0} \quad (4)$$

$$C(s) := \frac{a_r s^r + a_{r-1} s^{r-1} + ... + a_0}{b_r s^r + b_{r-1} s^{r-1} + ... + b_0} \quad (5)$$

Where $q := (n,d)$ and $n_i^- \leq n_i \leq n_i^+$ and $d_i^- \leq d_i \leq d_i^+$ for $i = 0,1,...,n$. Therefore, the closed loop characteristic polynomial is

$$G_{mf}(s) = \frac{G(s)}{1 + C(s)G(s)} = \frac{N_{mf}(s)}{D_{mf}(s)} \quad (6)$$

The goal here is to displace the poles of $G_{mf}(s)$, or equivalently, the roots of $D_{mf}(s)$ to a desired region $\delta(s)$, whose robust controller must be chosen to satisfy such desired region. Thus, let us now introduce a desired (or target) characteristic polynomial $\delta(s)$ of degree $n+r$, which is stable and has the desired set of the closed loop characteristic roots, as in (7).

$$\delta(s) = \delta_{n+r} s^{n+r} + \delta_{n+r-1} s^{n+r-1} + ... + \delta_0 \quad (7)$$

For robustness purposes, it is relaxed the requirements of attaining the target polynomial exactly and it is enlarged the target to a box in coefficient space containing the point representing the original desired characteristic polynomial. This corresponds to the choice of an interval desired polynomial family as the target rather than a single point. Therefore, suppose that (7) is described by the interval vector $[\delta_i] = \left[\delta_i^-, \delta_i^+\right]$ and consider that $D_{mf}(s) = \delta(s)$ to guarantee robust stability. According to Keel and Bhattacharyya (1999), for the controller design it is necessary and sufficient to solve the set of linear inequations constraints given in (8).

$$\begin{bmatrix} \delta_{n+r}^- \\ \delta_{n+r-1}^- \\ . \\ . \\ . \\ \delta_0^- \end{bmatrix} \leq \begin{bmatrix} b_r n_n + a_r d_n \\ b_r n_{n-1} + b_{r-1} n_n + a_r d_{n-1} + a_{r-1} d_n \\ . \\ . \\ . \\ b_0 n_0 + a_0 d_0 \end{bmatrix} \leq \begin{bmatrix} \delta_{n+r}^+ \\ \delta_{n+r-1}^+ \\ . \\ . \\ . \\ \delta_0^+ \end{bmatrix} \qquad (8)$$

Or equivalently

$$b_{i\,\min} \leq A_i x_c \leq b_{i\,\max} \qquad (9)$$

Where x_c is the vector with the robust controller parameters, to be optimized, $b_{i\,\min}$ and $b_{i\,\max}$ are respectively the linear inequalities constraints relative to lower and upper limits of the system. The array A_i corresponds to open loop plant coefficients.

2.3 *Chebyshev Theorem*

The Chebyshev Theorem says that it is possible to find the largest ball B of center x_c and maximum radius R, whose norm is Euclidean, which is contained in the politope P, described by the set of linear inequalities constraints. The ball center x_c is called Chebyshev Center, as shows the Figure 2 (Boyd and Vandenberghe, 2004).

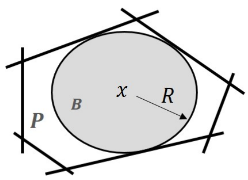

Figure 2: Largest ball B inscribed in P

When the set P is convex, the computing of x_c become a convex optimization problem. More specifically, suppose $P \subseteq R^n$ is defined by a set of convex inequalities, i.e., $P = \{\ a_i x \leq b_i, \quad i = 1, ..., m\ \}$. If $R \geq 0$, one can find x_c by solving the PL given in (10) (Boyd and Vandenberghe, 2004), with variables x and R.

$$\begin{array}{c} \text{maximize} \quad R \\ Subject \quad to \\ a_i x + R\|a_i\|_2 \leq b_i; \quad i = 1, ..., m \\ R \geq 0 \end{array} \qquad (10)$$

By assuming the (9) is linear or affine in the vector x_c and the PL techniques covered in detail in Boyd and Vandenberghe (2004), the robust pole placement technique reduces to a PL combined to the Chebyshev Theorem, as shown in (11).

$$\begin{array}{l} \max F(x_c, R) \\ \quad subject \quad to \\ A_i x_c + \|a\|\, R \leq b_i \end{array} \qquad (11)$$

Where:

$$A_c = \begin{bmatrix} A_i & \|a\| \\ -A_i & \|a\| \\ 0_{1\times i} & -1 \end{bmatrix}, \quad b_i = \begin{bmatrix} b_{i\,\max} \\ -b_{i\,\min} \end{bmatrix}, B_c = \begin{bmatrix} b_i \\ 0 \end{bmatrix} \qquad (12)$$

The vector $x = \begin{bmatrix} x_c & R \end{bmatrix}^T$ represents the parameter array to be optimized. $F(x_c, R)$ is an arbitrary linear function in x_c and R and $\|a\|$ is the Euclidean norm of coefficients of A_i.

3 Digital Controller Design Based on analogical Controller

Considering the analogical PID controller be defined by

$$u(t) = K_p e(t) + K_i \int e(t)dt + K_d \frac{de(t)}{dt} \qquad (13)$$

where K_c, K_i and K_d are PID gains and $e(t)$ is the system error. According to Åström and Hägglund (2006), the analogical PD controller is obtained by considering $K_i = 0$, i.e.,

$$u(t) = K_p e(t) + K_d \frac{de(t)}{dt} \qquad (14)$$

According to Åström and Hägglund (2006), the digital PD controller is obtained by considering that the derivative part in (14) would be the error variation within a range $dt := T_s$, where T_s is sampling time, such as

$$u(k) = K_p e(k) + \frac{K_d}{T_s}\left[e(k) - e(k-1)\right] \qquad (15)$$

So, rewriting (15), we have the control law of the digital PD controller, as shown in (16).

$$u(k) = \left[K_p + \frac{K_d}{T_s}\right] e(k) - \frac{K_d}{T_s} e(k-1) \qquad (16)$$

3.1 Motor DC System Modelling

The technique presented was experimentally evaluated in the position control of a DC motor with constant field electric current coupled to a load through two gears, as in (17), whose parameters values are presented in the Table 1 (Hamrick and Lordelo, n.d.).

$$\frac{\theta(s)}{E_a(s)} = \frac{K_t}{(L_a b_{1eq} + R_a J_{1eq})s^2 + (R_a b_{1eq} + K_t K_b)s} \tag{17}$$

Where $J_{1eq} = J_1 + \left(\frac{n_1}{n_2}\right)^2 \times (J_2 + J_l)$, $B_{1eq} = \left(\frac{n_1}{n_2}\right)^2 \times b_2$ and $J_l = 0.5\rho\pi r^4 h$. In the equation (17), the angular position $\theta(s)$ is considered the output and the armature voltage $E_a(s)$ is considered the input of the system.

Table 1: DC Motor Parameters

Parameters	Symbols	Values
Armature Resistance	R_a	33Ω
Armature Inductance	L_a	$0.00169H$
Motor Torque Constant	K_t	$0.0283Nm/A$
Electromotive force constant	K_b	$0.0283Vs/rad$
Motor inertia moment 1	J_1	$1.06 \times 10^2 kgm^2$
Motor inertia moment 2	J_2	$1.06 \times 10^2 kgm^2$
Motor shaft viscous friction 1	b_1	$5.8 \times 10^6 kgm^2/s$
Motor shaft viscous friction 2	b_2	$5.8 \times 10^6 kgm^2/s$
Number of gear teeth 1	n_1	100
Number of gear teeth 2	n_2	300
Load Disc Radius	r	$0.0254\ m$
Mass Density of Aluminium	ρ	$2702\ kg/m^3$
Load disk density	h	$0.00635m$

3.2 Robust PD Controller Design

By replacing the parameter values in the Table 1 into (17) and admitting a variations of $\pm10\%$ in the parameters R_a and h, such as $[R_a] = [30, 36]$ and $[h] = [0.00635, 0.00762]\, m$, we have

$$\frac{\theta(s)}{E_a(s)} = \frac{\left[b_0^-, b_0^+\right]}{s^2 + \left[a_1^-, a_1^+\right] s} \tag{18}$$

or equivalently

$$\frac{\theta(s)}{E_a(s)} = \frac{[294, 0376; 389, 1290]}{s^2 + [8, 3213; 11, 0124]\, s} \tag{19}$$

This paper aims to obtain the digital PD controller parameters from the analogical PD controller parameters, if it exists, so that the closed loop system satisfies the desired performance specification. So, by considering the analogical PD controller in (14) in the frequency domain be of the form

$$C(s) = K_p + K_d s \tag{20}$$

we define the analogical PD Controller parameters vector to be $x_c = \left[\begin{array}{cc} K_d & K_p \end{array}\right]$. The desired performance specifications for the transient response was an interval overshoot $[M_p] = [6, 14]\,\%$ centered in $M_{p_c} = 10\%$ and a peak time $T_p = 0.4s$. According to Åström and Hägglund (2006), for a linear second order system one obtains a interval damping factor $\zeta = [0.5305, 0.6671]$, centered in $\zeta_c = 0.5912$ and a interval undamped natural frequency $\omega_n = [9.2653, 10.5430]$, centered in $\omega_{n_c} = 9.7377$, resulting in a interval desired closed-loop characteristic polynomial (Lordelo and Fazzolari, 2014), given by

$$\delta(s) = s^2 + \left[\delta_1^-, \delta_1^+\right] s + \left[\delta_0^-, \delta_0^+\right] \tag{21}$$

Where $\left[\delta_1^-, \delta_1^+\right] = [9.8306, 14.0671]$ and $\left[\delta_0^-, \delta_0^+\right] = [85.8450, 111.1555]$. Now consider the closed loop system characteristic polynomial given by

$$D_{mf}(s) = s^2 + (a_1 + b_0 K_d)\, s + b_0 K_p \tag{22}$$

Note that our purpose is to select K_p and K_d such that $\Re\left(D_{mf}(s)\right) \subseteq \Re\left(\delta(s)\right)$, where $\Re(.)$ denotes the root space of (.) (Keel and Bhattacharyya, 1997). This purpose will be reached if $\mathcal{F}\left(D_{mf}(s)\right) \subseteq \mathcal{F}\left(\delta(s)\right)$, where $\mathcal{F}(.)$ denotes the family of polynomials. Therefore, replacing interval parameters by their vertices and after simplification by eliminating redundant inequalities, we construct the following set of linear inequalities that the controller parameter x_c should satisfy.

$$\begin{bmatrix} \delta_1^- - a_1^- \\ \delta_1^- - a_1^+ \\ \delta_1^- - a_1^- \\ \delta_1^- - a_1^+ \\ \delta_0^- \\ \delta_0^- \end{bmatrix} \leq \begin{bmatrix} b_0^- & 0 \\ b_0^- & 0 \\ b_0^+ & 0 \\ b_0^+ & 0 \\ 0 & b_0^- \\ 0 & b_0^+ \end{bmatrix} \begin{bmatrix} K_d \\ K_p \end{bmatrix} \leq \begin{bmatrix} \delta_1^+ - a_1^- \\ \delta_1^+ - a_1^+ \\ \delta_1^+ - a_1^- \\ \delta_1^+ - a_1^+ \\ \delta_0^+ \\ \delta_0^+ \end{bmatrix} \tag{23}$$

Thus, by replacing interval parameters values and solving the linear goal programming problem given in (11), it is obtained the robust analogical PD controller, given in (24),

$$C\,(s)\ = 0.1491 + 0.0076s \tag{24}$$

which satisfies the prescribed constraints on the system and desired performance parameters. Therefore, by using (16), it is obtained the robust digital PD controller, given by

$$u(k) = 0.3005e(k) - 0.1514e(k-1) \tag{25}$$

4 Results

4.1 Robust Stability Analysis

The robust stability analysis of the closed loop system it was made by means of the four Kharitonov polynomials stability analysis, given in the section 2.1, as shows the Table 2, whose roots are presented in the Figure 3.

Table 2: Four fixed Kharitonov polynomials

	Robust	Classic
$K_1(s)$	$s^2 + 10.55s + 43.85$	$s^2 + 10.06s + 82.87$
$K_2(s)$	$s^2 + 13.96s + 58.03$	$s^2 + 13.32s + 109.7$
$K_3(s)$	$s^2 + 10.55s + 58.03$	$s^2 + 10.06s + 109.7$
$K_4(s)$	$s^2 + 13.96s + 43.85$	$s^2 + 13.32s + 82.87$

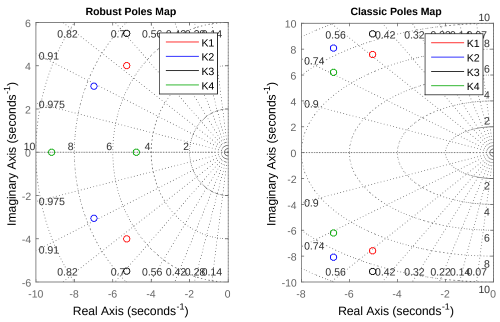

Figure 3: Four Kharitonov polynomials Pole Map

According to Table 2, it is noted that the two controllers provide robust stability, because it provide internal stability for all interval polynomial family, given in (19). Thus, the closed loop system is robustly stable because all its polynomials in this family have all their roots in the left hand side of the complex plane, as can be seen in the Figure 3.

4.2 Tracking Tests

For dynamic performance analysis, it was applied a step signal on the system input and compared with a classic PD controller designed by means of the classic pole placement technique. The Figure 4 to Figure 6 present the dynamic performance of closed loop system output and control signal with the two controllers (classic and robust) for the tracking asymptotic considering three operation points (lower-nominal-upper).

According to Figure 4 to Figure 6, the two controllers stabilize all interval polynomial family, satisfying the desired performance specification and the system output asymptotically tracks a step input in the face of parameter uncertainty. On the other hand, the robust controller provide better performance to parametric uncertain-

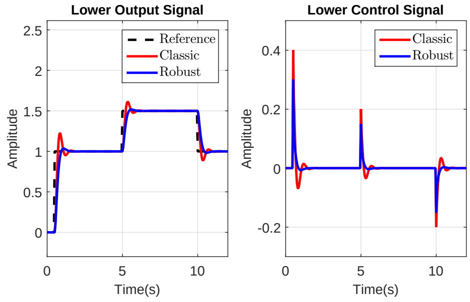

Figure 4: Dynamic performance in closed loop

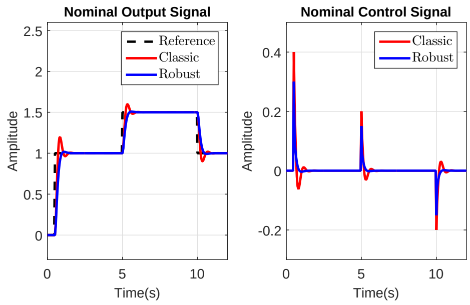

Figure 5: Dynamic performance in closed loop

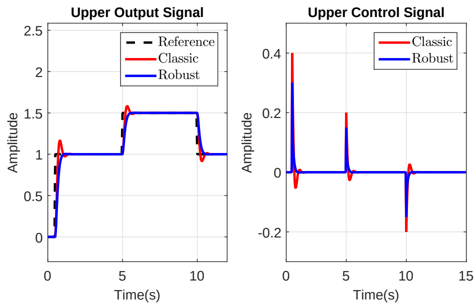

Figure 6: Dynamic performance in closed loop

ties in both the output and control signals, since it reached lower overshoot with lower control signal.

4.3 Performance Index Analysis

The performance of the two controllers is evaluated by "size" of an energy discrete sequence, which can be computed by a discrete approximation of the integral of a squared signal, given in (26), where W is a general vector that may assume to be the error $e(k)$ between the reference signal $y_r(k)$ and the measured output $y(k)$ and the control signal u for analysing of the control effort, respectively designated as ISE and ISU within this work (Silveira et al., 2016).

$$ISW = \sum_{k=1}^{n} (W^T W) T_s \tag{26}$$

The tables 3 and 4 present the performance index of the two controllers and prove the good performance of the robust controller, because this one

Table 3: Performance Index for Robust PD

	Output Signal	Control Signal
	ISE	***ISU***
Lower	4.2785	0.17284
Nominal	4.0783	0.16857
Upper	3.8905	0.16460

Table 4: Performance Index for Classic PD

	Output Signal	Control Signal
	ISE	***ISU***
Lower	3.6164	0.37864
Nominal	3.3793	0.36003
Upper	3.1567	0.34260

apresents lower control effort (ISU) to the three operation points, although presents bigger energy for tracking the reference signal $y_r(k)$, calculated by ISE. This was necessary for providing the lower overshoot presented.

5 Conclusions

In this paper, it was presented the robust controller design of fixed dynamic order for an uncertain system containing parameter uncertainty. The design it was realized by matching two design approaches to robust controllers which robustly place the desired closed loop poles, so that stability and robust performance are attained. The desired closed loop specifications considered were given in terms of a target performance vector representing a desired closed loop design and by enlarging the target from a fixed point set to an interval set the solvability conditions were relaxed and a solution was enabled.

The robust controller designed it was applied to tracking the angular position of a motor DC and compared with a PD controller designed by using classical pole placement technique. Through the results presented, it possible to note that the two controllers ensured the system robust stability and both presented a good performance when occur variations in the system parameters. The performance index analysis of the two controllers were presented to show the efficiency of robust controller. From these considerations, some conclusions can be highlighted:

- By means of the output signal and control signal analysis, it is possible to conclude that the system controlled by the robust PD controller presents smaller overshoot and performs smaller control effort.
- The technique presented devise a computationally simple linear programming approach that attempts to meet the desired closed loop specifications, because it does not require high computational efforts. The technique can be applied to industrial applications whose plant model presents parameter uncertainties. By converting from analogical to digital time, it was possible to obtain a digital controller with a good performance.
- To future works, is proposed a robust controller design law direct from in the digital domain. By this way, we hope to improve the performance of the controller.

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